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Lenses for Examining Students' Mathematical Thinking

A few years ago, a colleague shared a video from a first-year algebra class that he had observed. The video captured a class discussion about slopes of horizontal and vertical lines. At the beginning of the discussion, the teacher, Ms. Milner, asks, "How can we have a slope of zero?" Students respond in various ways; one student, Peter, explains that a horizontal line would have a slope of zero "because it's never moving up." Later, Milner asks about the slope of a vertical line, and another student, Alex, replies that because "it went . . . up and down and didn't move at all, it would be zero." Milner then asks the class about the slope of a line through the points (0, 0) and (0, 5). Peter says that "the slope is zero, because you subtract the change in x and the change in y. On the top there'd be zero and on the bottom there'd be 5, and any division problem that has zero in it has to be zero." Finally, Rafael disagrees and suggests that the slope is undefined because "in division there can't be a number over zero."

We were intrigued. What do these students understand about slope? What does Peter mean by "it's never moving up," and what does Alex mean when he says "it . . . didn't move at all"? To us, simply saying that Peter is initially correct, that Rafael is correct at the end, and that Alex is wrong does not capture the complexity of these students' thinking. Further, focusing on the surface-level correctness of their answers, rather than exploring the nuances of what the students are thinking, would not provide Milner with the information she needs to advance her students' understanding.

Of course, understanding students' thinking can be challenging, particularly in the moment of instruction (Sherin, Jacobs, and Philipp 2011). Students' ideas can be complex, and students may not always articulate their ideas clearly. The need to move the lesson forward and to attend to so many aspects of the classroom at once can leave a teacher with little time to think in depth about a student's comment. In addition, although the phrase "making sense of student thinking" is commonly used in teacher education and professional development, often little guidance is offered for doing so (Ball and Cohen 1999).

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For more than a decade, we have developed and studied video clubs as a context for helping mathematics teachers learn to attend more closely to the ideas that students raise in the classroom. In a video club, teachers meet to watch and discuss excerpts of video from their own classes (Sherin 2000). Our work with video clubs is driven by the belief that video can serve as a valuable resource for teachers, providing a window into classroom interactions within a time frame that allows for reflection. In addition, watching video excerpts with colleagues can provide opportunities to consider multiple perspectives on students' ideas.

We have studied video clubs with elementary school teachers (van Es 2009), middle school teachers (Sherin and Han 2004), and high school mathematics teachers (Walkoe 2013) and with both preservice and in-service teachers. Most teachers whom we have worked with have been involved in a single cycle of video club meetings over a school year or university term, although some have participated in video clubs for more than one year.

Our early work documented that, throughout a series of video club meetings, teachers' discussions shifted away from focusing primarily on issues of pedagogy and management to paying more attention to students' thinking as expressed in the video (Sherin and Han 2004). Further, initially teachers described (e.g., "she said . . .") and evaluated (e.g., "he was wrong") the thinking that they noticed; however, in the later video club meetings, teachers primarily interpreted (e.g., "he probably meant . . .") students' ideas. These findings were based on qualitative analyses of transcripts from video club meetings.

More recently, in collaboration with our colleague Elizabeth van Es, we investigated the effect of video club participation on teachers' classroom practices. We found that the increased focus on student thinking that teachers develop in the video club setting does influence their instruction. Teachers are able to bring an emphasis on better understanding student thinking back to their classrooms; in particular, they are more open to soliciting multiple student responses and more likely to follow up on those responses (van Es and Sherin 2010). These findings were the result of examining videotapes of whole-class discussions in participants' classrooms before and after the video club meetings. We coded these classroom discussions for the extent to which teachers attended to students' mathematical thinking as well as for the ways in which they did so during instruction.

In our most recent work, reported here, we wanted to better understand how video clubs support this kind of learning among teachers. To do so, we videotaped a series of video club meetings with secondary school mathematics teachers and then conducted qualitative analyses of those meetings. In particular, we highlighted and then categorized the different approaches that the facilitators and teachers used to make sense of the student ideas that were discussed. As a result of this analysis, we identified three lenses that teachers used repeatedly to make sense of student ideas expressed in the video clubs. Specifically, teachers frequently began by *taking an initial look* at a student's idea before *going deeper* and finally by *looking across* a single student's solutions to several different problems or across several students' solutions to a single problem. In our ongoing research, we are making these lenses explicit to video club facilitators, who report that they are a productive tool for supporting teachers in learning to interpret student thinking (Sherin et al. 2013).

***Martha*:** Let's see if we can decipher what Alex is saying.
***Allison*:** Well, he said, "It didn't move at all," so I'm wondering if he thinks whether horizontally or vertically, . . . if it doesn't move, that means zero . . . if one of the two directions doesn't move, then it's zero.
***Martha*:** So he's not making the distinction between xs moving or changing and ys moving or changing?
***Allison*:** He didn't say which one. He just said, "It went up and down and didn't move at all." So I'm assuming he meant it didn't move left and right. So since it didn't move in some direction, the slope was zero.
***Martha*:** Now where do you think he would get that idea from?
***Sheila*:** He may not have had a name for an undefined slope. I feel like he maybe is also mixing up horizontal and vertical, because he says, "because it's horizontal," and then he talks about the line going up and down.
***Rick*:** I think it actually kind of ties together with Peter, that it doesn't matter where the zero is. If you're not moving in one direction, the slope is zero. Kind of like how Peter says . . . if you have a slope of zero in the division problem, it's zero automatically.

Fig. 1 Teachers in a video club meeting discuss students' thinking.

THE THREE LENSES EXPLAINED

To better understand the three lenses, we will reconsider the slope discussion from Milner's class shared earlier but this time from the perspective of a video club discussion. This brief class discussion provides a number of student ideas about slope that teachers can productively explore in a video club meeting (Linsenmeier and Sherin 2009). **Figure 1** contains an excerpt from a video club discussion about Milner's class. Martha is the facilitator for this meeting; the other participants are teachers. In this excerpt, teachers are talking about the ideas shared by Alex and Peter.

Taking an Initial Look

An important first step in exploring a student's idea is to try to interpret the meaning of what the student is saying or doing (Cavey and Mahavier 2010; Lampert 2001). In particular, taking an initial look involves exploring the specifics of what a student may or may not understand. For example, when teachers discuss Peter's explanation—"because it's never moving up"—they can explore what he means when he says that the line is not moving. Is he confused about the slope formula or about division involving zero? In investigating these questions, the teachers move beyond simply concluding that Peter does not know the slope of a vertical line to trying to understand what he does know. Keep in mind that it can be valuable to

Taking an Initial Look	What is the student's idea? • What different comments does Peter make in the video? • What does he mean by "It's never moving up"?
	What misunderstanding does the student exhibit? • Does Peter believe that any division problem with a zero in it, anywhere, is equal to zero?
	What is reasonable about this idea? • What does Peter understand about division problems involving zero? • Is there a way to interpret Peter's comment about division with zero so that it makes sense?

Fig. 2 Taking an initial look at a student's thinking raises specific questions.

Going Deeper	Where did this idea come from? • Where do you think Peter would get that idea?
	Is there more than one way to understand what the student is saying? • How else can we interpret what Peter meant? • Do you think that Peter believes that a zero slope and an undefined slope are the same?

Fig. 3 Going deeper into a student's thinking prompts interpretation of the student's reasoning.

identify correct thinking in the middle of an idea that appears incorrect (Sherin 2002). **Figure 2** suggests some specific initial questions that teachers can ask about Peter's thinking in this video clip.

Alex's comments about slope also prove interesting to examine. At the beginning of the excerpt shown in **figure 1**, Martha prompts the group to "see if we can decipher what Alex is saying." Allison focuses on Alex's statement that "it didn't move at all" and wonders if not moving "means zero" to Alex. Notice that, to make sense of Alex's ideas, the teachers begin to hypothesize about Alex's reasoning. They are trying to determine what understanding he is using to justify his claims, what misunderstanding he might have about mathematics, and what about his reasoning is sensible. That is, they are taking an initial look.

Going Deeper

Once teachers have determined what a student seems to be saying and seems to understand, they might pursue the potential origins of the student's idea or whether there are alternative hypotheses about the student's reasoning. This is what we mean by going deeper.

Figuring out what might have prompted a student to think in a particular way can clarify the reasoning and is often an important step in trying to help a student reformulate his or her own thinking (Cohen 2004). When teachers consider the source of Peter's idea about the slope of a vertical line, they might ask whether he is building on Alex's comment or whether he is focusing on calculating slope using the two points provided by Milner (see **fig. 3**). As seen in the video club discussion of Alex's thinking (see **fig. 1**), the teachers go deeper when they respond to Martha's question, "Where do you think he would get that idea from?" Sheila offers a few alternatives, suggesting that perhaps Alex did not have "a name for undefined slope" or that he might have been "mixing up horizontal and vertical."

In addition, considering alternative interpretations of a student's idea can be useful for teachers (van Es 2009). When we are in the middle of instruction, we have a tendency to assume that we understand what a student is saying. Are we sure that our initial conclusions are correct, or might there be something else going on? Even when it is not possible to reach a definitive answer, practicing this skill of stepping back and questioning our own assumptions can help teachers be more open and more aware of the range of ways that students are thinking (Sherin and van Es 2009). **Figure 3** offers some questions that prompt teachers to think about multiple ways to interpret Peter's comments in the video clip.

Looking Across

As we work to interpret a specific student idea, it can be helpful to make connections to other ideas raised in class, either ideas expressed by other students in the class or other ideas expressed by the same student (Sherin and Han 2004). Trying to identify connections across multiple expressed ideas is what we mean by looking across.

In particular, exploring how ideas expressed by different students are related can provide additional insight into a particular student's comment. For example, both Peter and Alex talk about lines "not moving." Teachers might wonder whether the two students necessarily mean the same thing. In addition, do we get a more nuanced view of what Peter might have meant when we examine Alex's thinking? In the video club meeting, Rick looks across when he suggests that Alex's idea "kind of ties together with Peter." Rick explains that for both these students, "not moving in one direction" means that "the slope is zero" (see **fig. 1**).

In addition to looking across different students' ideas, considering how an individual student is thinking about multiple ideas can be productive. For instance, teachers can look at what Peter says about horizontal lines and compare his comment with what he says about vertical lines. In addition, teachers might ponder how a student would respond to new questions not posed in the video. For example, would Peter say that a horizontal line or a vertical line is steeper? Such questions can help teachers analyze what a student understands as well as prepare teachers to ask effective, probing questions during instruction.

See **figure 4** for some questions designed to support teachers as they examine Peter's thinking across contexts.

Looking Across	How does this idea compare with others? • Do Peter and Alex have the same idea about slope? • How does Rafael's understanding of slope compare with Peter's or Alex's?
	How might this student respond to another problem? • How would Peter talk about the slope of a line that is not vertical or horizontal? • How would Peter decide which of two given lines has a bigger slope?

Fig. 4 Looking at the student's thinking across contexts helps teachers analyze the student's understanding and develop effective questioning.

PUTTING THE LENSES TO USE

In presenting these three lenses, we have tried to take the abstract idea of "making sense of student thinking" and turn it into a more manageable and concrete task. Our work with teachers in video clubs shows that productive discussions arise when teachers ask questions across these three lenses. Moving forward, we are making the lenses explicit to facilitators and teachers to help guide teacher discussion of students' mathematical thinking.

Notice that in **figure 1** the video club participants use the lenses sequentially, starting with an initial look ("Let's see if we can decipher what Alex is saying"), then going deeper ("Now where do you think he would get that idea from?"), and finally looking across ("I think it actually kind of ties together with Peter"). We have also seen teachers have an extended discussion using a single lens or engage in several shorter cycles of an initial look, going deeper, and looking across during a single video club meeting.

Discussions of student thinking may also proceed in a less linear fashion, moving, for example, from an initial look directly to looking across or even from looking across back to going deeper or an initial look. This might happen when a desire to compare two students' thinking prompts us to look more closely at an individual student's idea. For example, we might not consider Alex's understanding of slope until we begin to wonder whether Peter is building on what Alex says.

We have found that using the lenses in different sequences does not constrain the depth of the discussions that take place. Instead, what is most important in supporting productive discussion of student thinking is for teachers to have opportunities to engage in all three strategies over time (Sherin 2007).

Also notice that in the discussion excerpt in **figure 1**, the facilitator, Martha, prompts the use of both taking

an initial look and going deeper, but it is a participant, Rick, who prompts the group to look across. We have found this shift to be a common one. In a new video club, it is most often the facilitator who prompts participants to apply the different lenses. Over time, however, teachers take on the three lenses themselves and require less specific guidance from a facilitator (Walkoe 2013).

TOOLS FOR UNDERSTANDING STUDENT THINKING

The three lenses presented here can be valuable tools for supporting teachers as they strive to make sense of student thinking. Although we have used these lenses primarily in the context of video clubs, they are equally applicable in other professional development settings that involve examining student work. We have found that teachers who use these strategies in a professional development setting often transfer this practice to their everyday teaching by applying the lenses in their classrooms. (Sherin and van Es 2009). As Laura reflected, "I wonder how many times I just hear 'zero' and I'm like, 'Yup, good job.' [Now] I wonder what they actually said . . . I need to [think about] this more."

Finally, although engaging in such detailed reasoning on your own is possible, teachers seem to value the opportunity to engage with peers as they use these lenses. As Sheila explained, "Having people to talk it over with, I feel like I'm getting more out of it than just [thinking about] a set of questions . . . on my own." Rick also emphasized that having the opportunity to hear different interpretations was important to him: "There were a couple of times where [my colleagues] interpreted it one way, and . . . I didn't even think of it that way, but [then I thought] 'Maybe that is what the kid meant.'"

Teachers interested in being part of a video club can start by inviting a few colleagues to watch and discuss a video from their classroom and then seeing whether those same colleagues would be willing to share a video of their own. Look for classroom episodes for which you think it would be productive to have your colleagues help you explore student ideas—that is, video clips in which student thinking is visible. Keeping the three lenses in mind can then support you and your colleagues as you begin to reflect on student thinking in a deep and meaningful way.

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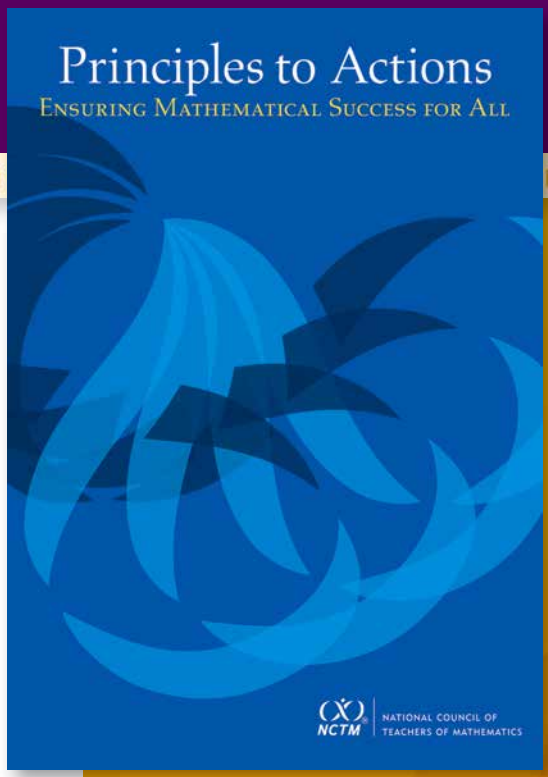

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